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LETTER TO THE EDITOR

Fluid penetration through a crack in a pressure gradient

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Abstract. We investigate invasion percolation fingers in a quenched medium in which the randomness has a gradient following a power law both in the direction of the flow and perpendicular to it. The first gradient corresponds to a pressure gradient, the second one to the density of microcracks that arises in a self-organized way around a large crack. We give an argument for the value of the fractal dimension as found in previous simulations. We calculate the roughness exponent of the fingers and find a value consistent with $\frac{1}{2}$ as predicted by the argument.

The penetration of fluids in porous media [1] is usually acompanied by the creation and propagation of major cracks through which this fluid will flow. This coupling between fluid transport and cracking has hardly ever been attacked theoretically, although it seems to be the main mechanism in technologically very important applications like hydraulic fracturing. It is the purpose of this letter to present a very simple model for this problem and its numerical analysis.

We want to consider a non-wetting fluid penetrating into a two-dimensional quenched disordered medium. The disorder is modelled by placing random mumbers z_i on the sites of a square lattice distributed according to a distribution $P(z_i)$. Fluid penetration into a random solid has often been described by invasion percolation. In particular, in [2], a model has been proposed in which a gradient is introduced by assuming the random numbers to be distributed as $P(z_i) \propto r^{-\alpha}$, with r the distance from the injection point. This graded distribution of random permeabilities models, to some degree, a pressure gradient. A numerical simulation of that model showed that for positive α the clusters are fractal with a fractal dimension of $d_f = 1.44 \pm 0.02$, independent of α. This result has not been understood yet. Here we will give an argument for this value.

The penetration of a fluid is, however, usually accompanied by the opening of a crack in the medium. It has been shown [3, 4] that brittle crack formation is accompanied by the formation of a cloud of microcracks around the main crack and that the density of crack surface generated by the microcracks decays like a power law as a distance ρ of the position of the main crack. We want to include this effect into the model proposed in [2] by assuming that $P(z_i) \propto \rho^{\beta}$. This will be implemented into our model by the following prescription:

We consider a square lattice of horizontal length L_1 and vertical height L_2 and with periodic boundaries in the vertical direction. At line j_0 on the left boundary we inject the fluid, i.e. we occupy this site. Then on each site of the first two columns a random permeability is placed distributed according to $P(z_i) \propto |\mathrm{mod}((j-j_0), L_2/2|^{\beta}$ where j is the line number of point i. Now the usual invasion percolation algorithm is applied: among all the sites on the surface of the finger, i.e. all the sites adjacent to the occupied sites, the one having the smallest permeability value is chosen and occupied. Each time the finger advances by one column the (random) permeabilities of the following column are chosen according to the distribution

$$P(z_i) \propto \left| \mathrm{mod}\left((j-j_t), \frac{L_2}{2} \right) \right|^{\beta} \times (k-1)^{-\alpha} \tag{1}$$

where k is the number of the column and j_t is the number of the line at which the finger just advanced by a new column. When the finger touches the right boundary the process is stopped.

In the described model the noise is quenched since once the random permeabilities are defined on a site they will not change any more. When $\beta=0$ we recover the model described in [2]. If $\beta>0$ the vertical width of the finger at a given column is confined by the power law gradient of permeabilities so that if L_2 is not chosen too small the finger can wind around the tube formed by the periodic boundary conditions making it insensitive to finite size effects in the vertical direction.

In the following we want to argue in favour of the fractal dimension $d_f \approx 1.44$ found for the model for $\beta=0$ (in two dimensions) [1]:

We define a dimensionless pressure $f=(p-p_c)/p_c$, which plays the same role as the occupation probability in random percolation. The existence of an external gradient induces a macroscopic anisotropy in the system: the fluid can penetrate the system in the direction of the macroscopic gradient (longitudinal direction) much easier than in the transverse direction (perpendicular to the macroscopic gradient). Therefore, it is not unreasonable to assume that the system is characterized by *two* correlation lengths ξ_L and ξ_T in the longitudinal and transverse directions, respectively. Thus, as $f \to 0$, one has

$$\xi_L \sim f^{-\nu_L} \qquad \xi_T \sim f^{-\nu_T}. \tag{2}$$

With these assumptions our problem becomes similar to directed percolation (for reviews and references see Kinzel [6] and Duarte *et al* [7]). However, aside from the fact that we are considering invasion percolation, we should emphasize the main difference between our model and directed percolation. The anisotropy in our model is *dynamically* induced: if we reverse the direction of the external gradient, we will still have macroscopic transport (in the direction of the macroscopic gradient), whereas the bias and anisotropy in directed percolation are *static* and fixed.

We can now calculate a fractal dimension for the finger. The number of sites N_s in it is the product of its volume $\xi_L \xi_T^{d-1}$ and the fraction of the sites of the lattice X^A that have been invaded by the fluid. If we assume that as $f \to 0$

$$X^A \sim f^{\beta} \sim \xi_L^{-\beta/\nu_L} \tag{3}$$

then, $N_s \sim \xi_L \xi_T^{d-1} \xi_L^{-\beta/\nu}$, and if for length scales $L \ll \xi_L$ we define the fractal dimension d_f by $N_s \sim \xi_L^{d_f}$, we obtain

$$d_f = 1 + \frac{1}{\nu_L} [\nu_T(d-1) - \beta]. \tag{4}$$

If we assume that the exponents defined above are the same as those defined for directed percolation, then, for $d=2$, $\nu_L \approx 1.734$, $\nu_T \approx 1.1$, and $\beta \approx 0.28$. These imply

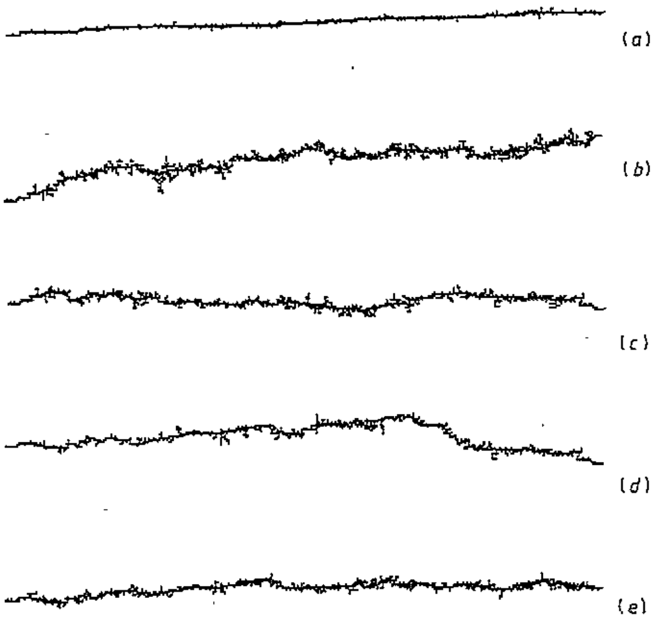

Figure 1. Invading fingers when they percolate through a system of $L_1=320$ ($L_2=41$) for (*a*) $\beta=2$, $\alpha=1$; (*b*) $\beta=0.5$, $\alpha=1$; (c) $\beta=0.7$, $\alpha=1$; (*d*) $\beta=0.7$, $\alpha=4$ and (*e*) $\alpha=1$ and an exponential distribution of the form $e^{0.5r}$ in the transverse direction.

that $d_f \simeq 1.47$, in good agreement with the simulation result of de Arcangelis and Herrmann [2].

Reference [2] also considered a directed invasion percolation process in a *radial* geometry, and obtained $d_f \simeq 1.29 \pm 0.03$. We argue that in this case the longitudinal direction is the same as in the previous case, but motion in the transverse direction is essentially a random walk. This implies that, $\xi_T \sim \xi_L^{1/2}$, i.e. $\nu_T = \nu_T = \nu_L/2$, which means that

$$d_f = 1 + \frac{d+1}{2} - \frac{\beta}{\nu_L}. \tag{5}$$

For $d=2$ we obtain $d_f \simeq 1.34$, in good agreement with the result of de Arcangelis and Herrmann [2].

Next let us present our numerical results for the case in which the quenched noise is given by (1). In figure 1 we see the invaded clusters for various parameters. If $\beta=2$ the invading finger is very straight (figure 1(a)) and for decreasing β (figure 1(b) and 1(c)) it has much denser sidebranches and stronger deviations from the line $j_0=$ constant given by the original injection point. The effect of α, on the contrary, is rather weak as seen when comparing figures 1(c) and 1(d). It is interesting to note that by taking an exponential distribution instead of a power law in the transverse direction no qualitative difference is noted as can be seen from figure 1(e).

We calculate the width w of the invasion cluster through

$$w = \frac{1}{N_s} \sum_{1}^{N_s} |j - j_0| \tag{6}$$

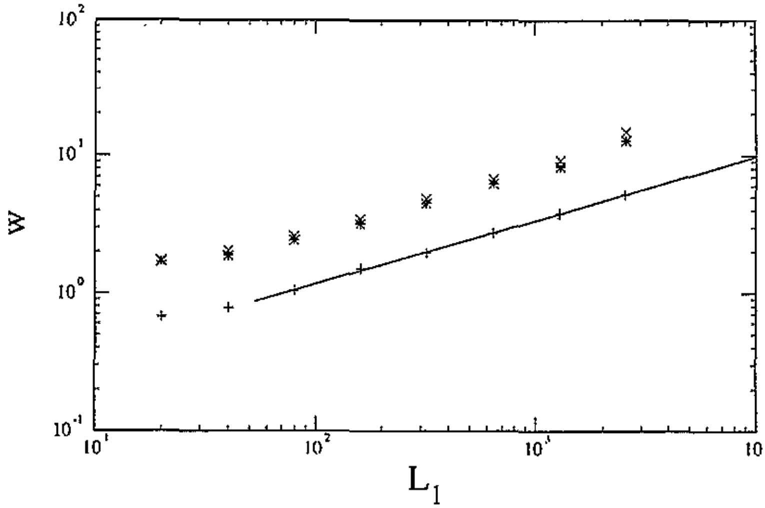

Figure 2. Log–log plot of the width w of the finger as a function of its length L_1 for $L_2=23$, $\alpha=1$ and $\beta=1$ ($*$), $\beta=2$ ($+$) and an exponential distribution $e^{0.5r}$ ($\times$). The straight line is a linear fit through the last six data points and has a slope of 0.46.

where N_s is the total number of sites in the finger. Then w is averaged over 1000 different samples. In figure 2 we see a log–log plot of the width w as a function of L_1. In all cases the data fall asymptotically on straight lines of slope a little below 0.5. Since the curves, however, have a slight upward curvature the resulting roughness exponent ζ defined through

$$w \sim L_1^{\zeta} \tag{7}$$

could well be $\frac{1}{2}$. From figure 2 we also see that the detailed distribution of quenched permeabilities in the transverse direction has no noticeable effect on the roughening exponent. Even an exponential distribution gives the same result.

The numerical data suggest that the penetrating fluid essentially performs a random walk in the transverse direction ($\zeta=\frac{1}{2}$). The confining distribution given by the power law distribution of microcracks only has an effect on the amplitude of the width of the clusters but not on the roughening exponent.

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